REMINDER:

① GEOMETRIC SERIES TEST $\sum_{i=1}^{\infty} ar^{i-1}$ CONV IF $|r|<1$

(IS EACH TERM A FIXED MULTIPLE OF PREVIOUS TERM?) DIV IF $|r| \geq 1$

② DIVERGENCE TEST

IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_{i=1}^{\infty} a_i$ DIV

11.3

REVISIT $\sum_{i=1}^{\infty} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots$

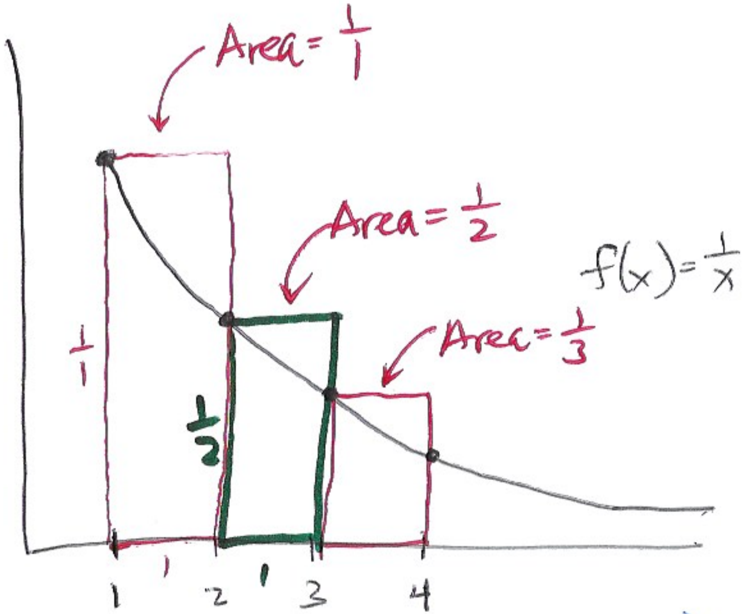
GEO TEST DOESN'T APPLY

$\frac{\frac{1}{2}}{\frac{1}{1}} = \frac{1}{2} \neq \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

CONSIDER $f(x) = \frac{1}{x}$, $x \geq 1$

DIV TEST DOESN'T APPLY

$\lim_{n\to\infty} \frac{1}{n} = 0$



IF $\int_1^{\infty} \frac{1}{x}\,dx$ DIVERGES ⟵ TRUE

IE. $\int_1^{\infty} \frac{1}{x}\,dx \longrightarrow \infty$

THEN $\sum_{i=1}^{\infty} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots$

DIVERGES

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{N\to\infty} \int_1^{N} \frac{1}{x}\,dx = \lim_{N\to\infty} \ln|x| \Big|_1^N$$

$$= \lim_{N\to\infty} (\ln|N| - 0) = \lim_{N\to\infty} \ln|N| = \infty$$

DNE

SO, $\sum_{i=1}^{\infty} \frac{1}{i}$ DIV (INTEGRAL TEST)

CONSIDER $\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

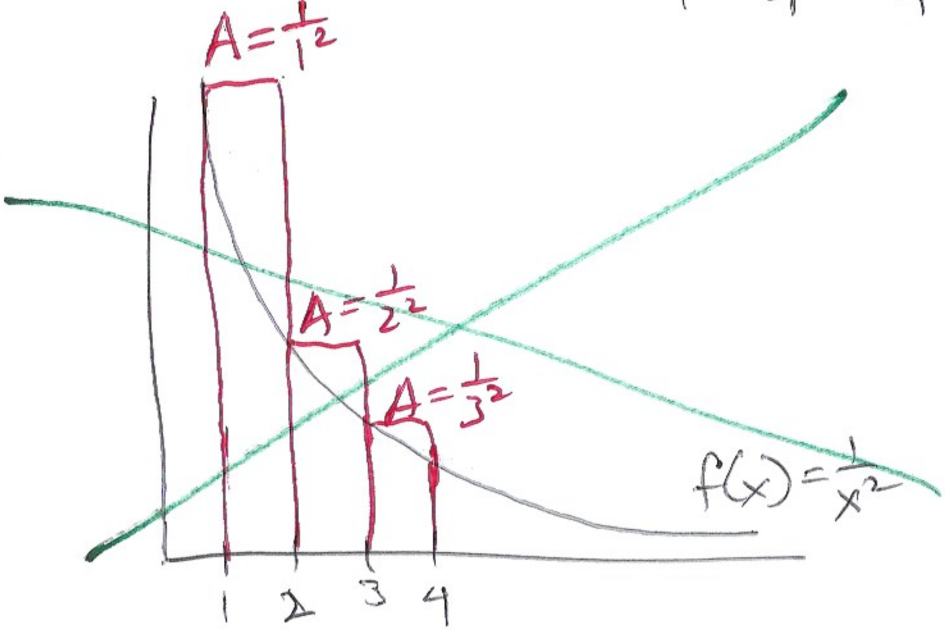
$= 1 + \frac{1}{4} + \frac{1}{9} + \dots$

GEO TEST DOESN'T APPLY

$\frac{1}{4} \div 1 = \frac{1}{4} \neq \frac{1}{9} \div \frac{1}{4} = \frac{4}{9}$

DIV TEST DOESN'T APPLY

$\lim_{n \to \infty} \frac{1}{n^2} = 0$



$\int_1^{\infty} \frac{1}{x^2} dx$

$= \lim_{N \to \infty} \int_1^N \frac{1}{x^2} dx$

$= \lim_{N \to \infty} -\frac{1}{x} \Big|_1^N$

$= \lim_{N \to \infty} \left(-\frac{1}{N} - -1\right)$

$= \lim_{N \to \infty} \left(1 - \frac{1}{N}\right)$

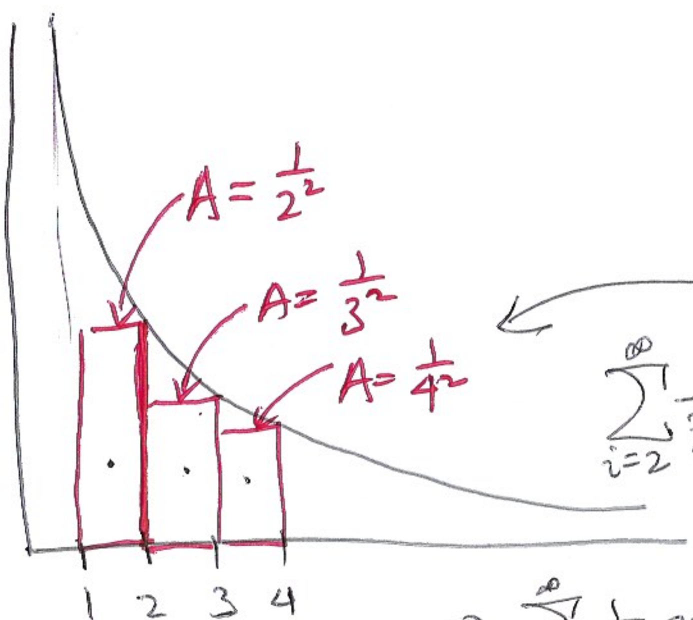
$= 1 - 0 = 1$ CONV

$\int x^{-2} dx$

$= \frac{1}{-1} x^{-1} + \cancel{C}$

$= -\frac{1}{x}$

AREA UNDER $\frac{1}{x^2}$ ON $[1, \infty)$ IS FINITE



AREA OF RED RECTANGLES IS FINITE

$\sum_{i=2}^{\infty} \frac{1}{i^2} = \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$ CONV

SEQUENCE OF PARTIAL SUMS INCR

+ BOUNDED ABOVE BY $\int_1^{\infty} \frac{1}{x^2} dx$

SO, $\sum_{i=2}^{\infty} \frac{1}{i^2}$ CONV $\leftarrow$ SO, SEQ OF PARTIAL SUMS CONV

SO, $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i^2} = \underbrace{\frac{1}{1^2} + \underbrace{\sum_{i=2}^{\infty} \frac{1}{i^2}}_{\text{CONV, IS A NUMBER}}}_{\text{IS A NUMBER}}$ CONV (INTEGRAL TEST)

~~CONSIDER~~

INTEGRAL TEST

IF $f(x)$ IS POSITIVE, DECREASING AND CONTINUOUS ON $[1,\infty)$

AND $a_n = f(n)$

THEN $\int_1^\infty f(x)\,dx$ CONV IF AND ONLY IF $\sum_{i=1}^{\infty} a_i$ CONV

(ALSO MEANS THAT IF $\int_1^\infty f(x)\,dx$ DIV, THEN $\sum_{i=1}^{\infty} a_i$ DIV)

eg.

$\displaystyle\sum_{n=1}^{\infty} n e^{-n}$

$ne^{-n} > 0$, $f(x) = xe^{-x}$ IS CONT ON $[1,\infty)$

$f'(x) = e^{-x} - xe^{-x} = \underbrace{(1-x)}_{<0 \text{ ON } (1,\infty)}\underbrace{e^{-x}}_{>0} < 0$ ON $(1,\infty)$

SO f IS DECR ON $[1,\infty)$

CAN USE INTEGRAL TEST